RESIDUAL STRUCTURAL STRESSES IN REINFORCED SHELLS OF ROTATION

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A typical feature of structures made of composite materials is presence in them of residual stress and strain fields which form as a result of the manufacturing production process and which affect the strength of a finished product and deviation of its shape from that initially prescribed. The problem of forming these fields in layered reinforced cylindrical bodies prepared by the winding method have recently been studied in detail (see [1]). There has been much less solution of the problem for structures which differ from cylindrical [2]. In the existing works residual stresses in a composite material are determined at the microlevel and formation of residual stresses in the reinforcement and binder (structural stresses) are not considered. In addition, experiments on simple unidirectionally reinforced specimens [3] show that these stresses, which are caused in particular by thermal and chemical shrinkage of the binder, may reach considerable values with complete absence of microstresses. A method is suggested in this work for finding the residual stress—strained state (also including structural stresses) in reinforced axisymmetrical shells prepared by winding or forming.

1. The process of preparing shells is presented as a sequence of five stages [4]; formation of a semifinished shell by lay-up (winding) of unidirectionally reinforced layers on a mandrel, heating the semifinished product together with the mandrel to the binder polymerization temperature and its formation of an external force effect, polymerization of the binder, cooling the finished shell on the mandrel, and removing the mandrel.

We make the following assumptions: 1) as a result of the viscosity of unhardened binder structural stresses which form in the composite towards the instant of a changeover of the binder to an elastic phase are entirely relaxed; therefore in the first two stages we consider the binder as an ideal fluid, and in the rest of the stages as an isotropic solid for which the Duhamel—Neumann law is valid; this law is valid for the reinforcing material in all stages; 2) polymerization is an instantaneous change in the properties of the binder from the liquid to the solid phase; 3) in view the thin wall of the shell the temperature field during the whole process is uniform through the thickness and the change in stresses in the reinforcing skeleton caused by filtration of binder and movement of the polymerization front may be ignored [5]; 4) the reinforcement and hardened binder are in a plane stressed state and are ideally fastened to each other.

With these assumptions the production process may be conditionally broken down into two stages in relation to the aggregate condition of the binder: the first includes the first two stages of the process and the second includes the final three stages. We introduce into planes of a unidirectionally reinforced layer orthogonal axes (1, 2) connecting the first axis with the reinforcement direction. By using a fibered model as a basis of assumption 1, we obtain equations of state for the unidirectional composite with unhardened binder in the form

$$\Delta\sigma_{1(k)} = \xi' E'(\Delta\varepsilon_{1(k)} - \alpha'\Delta t_k),\quad \Delta\sigma_{2(k)} = \Delta\sigma_{12(k)} = 0\ (k = 1, 2), \tag{1.1}$$

and stresses in the reinforcement are connected with increments of macrostresses by the relationship

$$\sigma'_{1(k)} = \sigma'_{1(k-1)} + \Delta\sigma_{1(k)}/\xi',\ \sigma'_{2(k)} = \sigma'_{12(k)} = 0. \tag{1.2}$$

Here and subsequently $\Delta\sigma_1(k) = \sigma_1(k) - \sigma_1(k-1)$; $\Delta\varepsilon_1(k) = \varepsilon_1(k) - \varepsilon_1(k-1)$; $\sigma_1(0) = \sigma_0$; $\sigma_2(0) = \sigma_{12}(0) = 0$; $\varepsilon_1(0) = \varepsilon_2(0) = 0$; $\sigma_1'(0) = \sigma_0/\xi'$; $\sigma_2'(0) = \sigma_{12}(0)' = 0$; $\Delta t_k = t_k - t_{k-1}$; $t_0 = t_1$; k is stage number; σ_1, σ_2, σ_{12}, ε_1 are average microstresses and macrostrains in the composite; a prime indicates corresponding average values relating to the

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reinforcement; E', ν', α' are Young's modulus, Poisson's ratio, and thermal expansion; ξ' is degree of reinforcement; σ_0 is prescribed initial stress in the layer during lay-up of the semifinished product (e.g., due to tension with power winding); t_k is temperature of the semifinished product at the end of the k-th stage.

By using an approach in [6] suggested for a composite with a hardened binder and considering additional shrinkage and thermal deformation, and also prior stressing in structural elements, we find equations of state for a unidirectional composite in the steps of the second stage (k = 3, 4, 5):

$$\Delta\{\sigma_{12(k)}\} = [g_k](\Delta\{\varepsilon_{12(k)}\} - \{\alpha_{12(k)}\}\Delta t_k - \{\lambda_{12(k)}\}\delta_{3k}\varepsilon_0), \quad (1.3)$$

and structural stresses are connected with increments of macrostresses by the relationships

$$\begin{gathered}\sigma'_{2(k)} = \sigma''_{2(k)} = \sigma''_{2(k-1)} + \Delta\sigma_{2(k)},\ \sigma'_{12(k)} = \sigma''_{12(k)} = \sigma''_{12(k-1)} + \Delta\sigma_{12(k)},\\ \sigma'_{1(k)} = \sigma'_{1(k-1)} + (E'\Delta\sigma_{1(k)} + \xi''\lambda\Delta\sigma_{2(k)})/E_1 + \xi''\sigma_{(k)},\\ \sigma''_{1(k)} = \sigma''_{1(k-1)} + (E''\Delta\sigma_{1(k)} - \xi'\lambda\Delta\sigma_{2(k)})/E_1 - \xi'\sigma_{(k)},\\ \sigma_{(k)} = E'E''\left[(\alpha'' - \alpha')\Delta t_k + \delta_{3k}\varepsilon_0\right].\end{gathered} \quad (1.4)$$

Here $\Delta\{\sigma_{12(k)}\}$, ..., $\{\lambda_{12(k)}\}$ are matrix columns (index k is omitted):

$$\Delta\{\sigma_{12}\} = \{\Delta\sigma_1,\ \Delta\sigma_2,\ \Delta\sigma_{12}\}^{\mathrm{T}},\ \Delta\{\varepsilon_{12}\} = \{\Delta\varepsilon_1,\ \Delta\varepsilon_2,\ \Delta\varepsilon_{12}\}^{\mathrm{T}},$$
$$\{\alpha_{12}\} = \{\alpha_1,\ \alpha_2,\ 0\}^{\mathrm{T}},\ \{\lambda_{12}\} = \{\lambda_1,\ \lambda_2,\ 0\}^{\mathrm{T}};$$

$[g_k]$ is a 3 × 3 symmetrical matrix whose elements have the form

$$\begin{gathered}g_{11}^{(k)} = \nu E_1,\ g_{12}^{(k)} = \nu E_1\nu_{12},\ g_{22}^{(k)} = \nu E_2 \quad (k = 3, 4, 5),\\ g_{13}^{(k)} = g_{23}^{(k)} = 0,\ g_{33}^{(k)} = G_{12}, \quad \nu = (1 - \nu_{12}\nu_{21})^{-1},\ E_1 = \xi'E' + \xi''E'',\\ E_2 = E_1E'E''\left[E_1(\xi'E' + \xi''E'') - \xi'\xi''\lambda^2\right]^{-1},\ \nu_{21} = \xi'\nu' + \xi''\nu'',\\ G_{12} = G'G''(\xi'G'' + \xi''G')^{-1},\ \lambda = \nu'E' - \nu''E',\ \alpha_1 = (\xi'E'\alpha' + \xi''E''\alpha'')/E_1,\\ \lambda_1 = \xi''E''/E_1,\ \xi'' = 1 - \xi',\ \alpha_2 = \xi'\alpha' + \xi''\alpha'' + \xi'\xi''\lambda(\alpha' - \alpha'')/E_1,\\ \lambda_2 = \xi''(E_1 - \xi'\lambda)/E_1;\end{gathered} \quad (1.5)$$

values labelled with two primes relate to the binder; G is shear modulus; ε_0 is shrinkage strain for the binder in the solid phase; δ_{3k} is Kronecker symbol.

By using (1.2) and (1.5) we write the equations of state for a unidirectional composite the same for both stages in axes (s, θ) turned in the plane of the layer with respect to axis (1, 2) by arbitrary angle ψ:

$$\begin{gathered}\Delta\{\sigma_{s\theta(k)}\} = [B_k]\Delta\{\varepsilon_{s\theta(k)}\} - \{\alpha_{s\theta(k)}\}\Delta t_k - \{\lambda_{s\theta(k)}\}\delta_{3k}\varepsilon_0,\\ ([B_k],\ \{\alpha_{s\theta(k)}\},\ \{\lambda_{s\theta(k)}\}) = [T_1][g_k]([T_1]^{\mathrm{T}},\ \{\alpha_{12(k)}\},\ \{\lambda_{12(k)}\}),\end{gathered} \quad (1.6)$$

where $[T_1]$ is matrix for transformation of stress tensor components with rotation [7]. In (1.6) elements of matrices $[g_k]$, $\{\alpha_{12(k)}\}$, $\{\lambda_{12(k)}\}$ are determined for the first stage (k = 1,2) by the relationship

$$\begin{gathered}g_{11}^{(k)} = \xi'E',\ g_{12}^{(k)} = g_{22}^{(k)} = g_{23}^{(k)} = g_{33}^{(k)} = 0,\ \alpha_{1(k)} = \alpha',\\ \alpha_{2(k)} = 0,\ \lambda_{1(k)} = \lambda_{2(k)} = 0,\end{gathered} \quad (1.7)$$

and for the second (k = 3, 4, 5) by relationships (1.6). It is noted that from (1.3) and the transformation equation with rotation $\Delta\{\varepsilon_{12(k)}\} = [T_1]^{\mathrm{T}}\Delta\{\varepsilon_{s\theta(k)}\}$ the equality

$$\Delta\{\sigma_{12(k)}\} = [g_k]([T_1]^{\mathrm{T}}\Delta\{\varepsilon_{s\theta(k)}\} - \{\alpha_{12(k)}\}\Delta t_k - \{\lambda_{12(k)}\}\delta_{3k}\varepsilon_0), \quad (1.8)$$

follows which together with (1.2) or (1.4) establishes the dependence between structural stresses and increments of macrostrains for the composite in all stages of the process.

2. We consider a shell of rotation formed by a layer of pairs of unidirectionally reinforced layers symmetrical with respect to te central surface. Layers of one of the pairs are identical, and the reinforcement trajectory comprise angles $\pm\psi_m$ (m = 1, ..., n, n is number of pairs of layers) with the direction of the meridian of the central surface. In order to describe the stress—strained state of the shell during the manufacturing process we assume that: 1) for all of the bundle of layers the Kirchhoff—Love hypotheses are valid; 2) equations of state for elementary layers have the form (1.6); 3) the shell is in an axi-

symmetrical moment-free state and in contact with the mandrel and forming device without friction. Compliance of the mandrel compared with compliance of the semifinished product is ignored. The equations of shell theory [8] for the k-th stage are written as

$$r dT_{s(k)}/ds + (T_{s(k)} - T_{\theta(k)}) dr/ds = 0, \tag{2.1}$$
$$T_{s(k)}/R_1 + T_{\theta(k)}/R_2 = p_k, \; p_k = p_k^+ + p_k^-,$$
$$T_{s(k)} = T_{s(k-1)} + C_{11}^{(k)}\Delta\varepsilon_{s(k)} + C_{12}^{(k)}\Delta\varepsilon_{\theta(k)} - C_{13}^{(k)}\Delta t_k - C_{14}^{(k)}\delta_{3k}\varepsilon_0,$$
$$T_{\theta(k)} = T_{\theta(k-1)} + C_{12}^{(k)}\Delta\varepsilon_{s(k)} + C_{22}^{(k)}\Delta\varepsilon_{\theta(k)} - C_{23}^{(k)}\Delta t_k - C_{24}^{(k)}\delta_{3k}\varepsilon_0,$$
$$\Delta\varepsilon_{s(k)} = d(\Delta u_k)/ds + \Delta w_k/R_1, \; r\Delta\varepsilon_{\theta(k)} = \Delta u_k dr/ds + w_k \sin\varphi,$$
$$\Delta\varepsilon_{s\theta(k)} = 0, \; u_k = u_{k-1} + \Delta u_k, \; w_k = w_{k-1} + \Delta w_k, \quad u_0 = w_0 = 0.$$

Here

$$(T_{s(0)}, T_{\theta(0)}) = 4\sum_{m=1}^{n} (\sigma_{s(0)}, \sigma_{\theta(0)})_m h_m;$$
$$(C_{1i}^{(k)}, C_{22}^{(k)}, C_{13}^{(k)}, C_{23}^{(k)}, C_{14}^{(k)}, C_{24}^{(k)}) = 4\left[\sum_{m=1}^{n} (B_{1i}, B_{22}, \alpha_s, \alpha_\theta, \lambda_s, \lambda_\theta)_m h_m\right]_{(k)}$$
$$(i = 1, 2);$$

$T_{s(k)}$, $T_{\theta(k)}$, u_k, w_k are forces and displacements in the shell at the end of the k-th stage; r, R_1, R_2 are radius of rotation and principal curvatures of the central surface; s is meridian arc length; φ is angle between the normal to the central surface and the axis of rotation; $p_k^{\pm}$ are normal components of loads operating on the outer (+) and inner (−) surfaces of the shell in the k-th stage; h_m is thickness of the unidirectional layer in the m-th pair of the bundle of layers.

At high edges of the shell s = 0, s = s_1 it is necessary to prescribe for one value from each of values

$$(T_{s(k)}(0), \; u_k(0)), \; (T_{s(k)}(s_1), \; u_k(s_1)). \tag{2.2}$$

In addition, the stiffness condition for the mandrel superimposes a limitation on deflection:

$$w_k \geqslant 0 \quad \text{or} \quad w_k \leqslant 0. \tag{2.3}$$

The first limitation occurs when the mandrel is placed from the direction of the inner surface of the shell, and the second when it is from the direction of the outer surface. In the first case $p_k^+(s)$ is external forming pressure, and $p_k^-(s)$ is reaction to the mandrel, and in the second case functions $p_k^+(s)$, $p_k^-(s)$ change meaning. It is also noted that from the contact conditions $p_k^+ \le 0$, $p_k^- \ge 0$ and equality $p_k = p_k^+ + p_k^-$ it follows that

$$p_k^+ \leqslant p_k, \; p_k^- \geqslant p_k. \tag{2.4}$$

Relationships (2.1)-(2.3) with k = 1, ..., 4 and (2.1), (2.2) with k = 5 form a recurrent sequence of boundary problems for determining in each of the stages functions $T_{s(k)}$, $T_{\theta(k)}$, $\Delta\varepsilon_{s(k)}$, $\Delta\varepsilon_{\theta(k)}$, u_k, w_k, and with k = 1, ..., 4 the resultant external pressure on the shell $p_k(s)$ is also the function sought, but with k = 5 (mandrel removed) $p_5(s) = 0$. This sequence of problems for the stress-strained state of a shell in the concluding stages of the production process found as a result of the solution will be the residual state sought in the finished shell. Corresponding structural residual stresses are found from (1.4) and (1.8) taking account of (1.5).

In view of linearity of the problem in question this sequence may be reduced to solution of two boundary problems relating to two stages of the process. In each stage as a power load there is the load in the last step of the stages, and as increments there are the differences in corresponding functions at the start and end of the stages.

We carry out solution of these problems. In the first stage we take k = 2 and $\Delta t_2 = t_2 - t_0$, $\Delta\varepsilon_{s(2)} = \varepsilon_{s(2)} - \varepsilon_{s(0)} = \varepsilon_{s(2)}$, $\Delta\varepsilon_{\theta(2)} = \varepsilon_{\theta(2)} - \varepsilon_{\theta(0)} = \varepsilon_{\theta(0)}$, $\Delta u_2 = u_2 - u_0 = u_2$, $\Delta w_2 = w_2 - w_0 = w_2$. For definiteness we consider the case when in the forming stages the semifinished product is entirely butted against the mandrel, i.e., condition (2.3) has the form

$$w_2 = 0. \tag{2.5}$$

Then by excluding in (2.1) forces and strain increments, and considering (2.5), we obtain

$$a_1 d^2u_2/ds^2 + a_2 du_2/ds + a_3 u_2 = a_4; \tag{2.6}$$

$$p_2 = b_1 du_2/ds + b_2 u_2 + b_3, \tag{2.7}$$

where functions $a_1(s)$, ..., $b_3(s)$ are expressed in terms of $r(s)$, $C_{ij}^{(2)}$, $C_{i3}^{(2)}$, $C_{i4}^{(2)}$, $T_{s(0)}$, $T_{\theta(0)}$ (i, j = 1, 2) and their derivatives with conditions (1.7). The general solution of Eq. (2.6) written as $u_2 = c_1 f_1(s) + c_2 f_2(s) + f_3(s)$ ($f_1(s)$, $f_2(s)$ is linearly independent partial solutions of the homogeneous equation, $f_3(s)$ is partial solution of Eq. (2.6)). Constants c_1 and c_2 are determined from boundary conditions (2.2) in which in prescribing conditions at the edges of the shell it should be assumed that

$$T_{s(2)} = C_{11}^{(2)}\, du_2/ds + C_{12}^{(2)} u_2\, dr/ds + T_{s(0)} - C_{13}^{(2)}\Delta t_2.$$

For displacement u_2 from (2.1) we find $\varepsilon_{s(2)}$, $\varepsilon_{\theta(2)}$, $T_{s(2)}$, $T_{\theta(2)}$ and subsequently from (1.2) and (1.8) we find stresses in the reinforcement $\{\sigma_{12(2)}{}^1\}$ at the end of the first stage.

It is noted that if formation of a shell is carried out by uniform pressure $p_2{}^+ = \text{const}$, then from the first inequality (2.4) considering (2.7) we have a limitation on $p_2{}^+$ which when fulfilled provides a condition of close butting of the semifinished product to the mandrel (2.5) in the formation process:

$$p_2^+ \leqslant p_*^+ = \min_{0 \leqslant s \leqslant s_1} p_2(s). \tag{2.8}$$

Similarly with formation pressure $p_2{}^-$

$$p_2^- \geqslant p_*^- = \max_{0 \leqslant s \leqslant s_1} p_2(s).$$

In the second stage we assume that k = 5 and $\Delta t_5 = t_5 - t_2$, $p_5 = 0$, $\Delta\varepsilon_{s(5)} = \varepsilon_{s(5)} - \varepsilon_{s(2)}$, $\Delta\varepsilon_{\theta(5)} = \varepsilon_{\theta(5)} - \varepsilon_{\theta(2)}$, $\Delta u_5 = u_5 - u_2$, $\Delta w_5 = w_5 - w_2 = w_5$. By using these dependences in solving Eqs. (2.1) we find the stress-strained state of the shell after the end of the production process:

$$\begin{gathered} T_{s(5)} = c_3(r\sin\varphi)^{-1},\ T_{\theta(5)} = -c_3(R_1\sin^2\varphi)^{-1}, \\ \Delta T_{s(5)} = T_{s(5)} - T_{s(2)},\ \Delta T_{\theta(5)} = T_{\theta(5)} - T_{\theta(2)}, \\ \Delta\varepsilon_{s(5)} = a_{11}\Delta T_{s(5)} + a_{12} T_{\theta(5)} + a_{13}\Delta t_5 + a_{14}\varepsilon_0, \\ \Delta\varepsilon_{\theta(5)} = a_{12}\Delta T_{s(5)} + a_{22}\Delta T_{\theta(5)} + a_{23}\Delta t_5 + a_{24}\varepsilon_0, \\ u_5 = u_2 + \sin\varphi\left[c_4 + \int_{\varphi_0}^{\varphi} \frac{1}{\sin\varphi}\left(R_1\Delta\varepsilon_{s(5)} - R_2\Delta\varepsilon_{\theta(5)}\right)d\varphi\right], \\ w_5 = R_2\Delta\varepsilon_{\theta(5)} - \Delta u_5 \operatorname{ctg}\varphi,\ a_{11} = C_{22}^{(5)} D^{-1},\ a_{12} = -C_{12}^{(5)} D^{-1},\ a_{22} = C_{11}^{(5)} D^{-1}, \\ D = C_{11}^{(5)} C_{22}^{(5)} - \left(C_{12}^{(5)}\right)^2,\ a_{ij} = a_{1i} C_{1j}^{(5)} + a_{2i} C_{2j}^{(5)} \quad (i = 1, 2, j = 3, 4) \end{gathered} \tag{2.9}$$

(c_3 and c_4 are integration constants determined from boundary conditions (2.2)). By substituting values $\Delta\{\varepsilon_{s\theta(5)}\}$ from (2.9) in (1.8) and (1.4), we obtain residual structural stresses $\{\sigma_{12(5)}\}_m$, $\{\sigma_{12(5)}{}''\}_m$ (m = 1, ..., n) in layers of the finished shell.

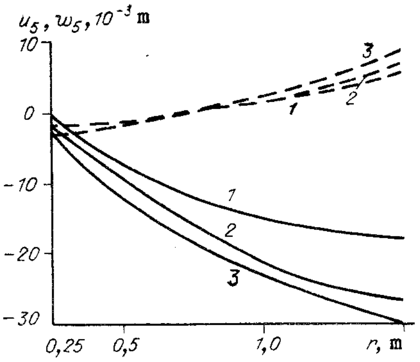

Fig. 1

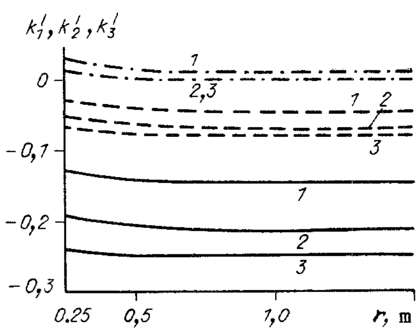

Fig. 2

3. As an example we give the results of calculations by the algorithm described above for residual displacements and structural stresses in a glass-plastic shell prepared by forming with uniform pressure p_2^+ on the mandrel. The surface of the mandrel has the shape of a paraboloid of rotation with an equation of the meridian $r = \sqrt{az}$ ($r_0 \le r \le r_1$) (z is coordinate along the mandrel axis). The shell is composed of six pairs ($n = 3$) of uniform unidirectionally reinforced layers. Reinforcement angles in pairs: $\psi_1 = 60°$, $\psi_2 = 30°$, $\psi_3 = 0$. The thickness of the elementary layers in the pairs $h_1 = h_2 = 10^{-4}$ m, $h_3 = 0.5 \cdot 10^{-4}$ m. Reinforcement and binder in a unidirectional layer have the characteristic : $E' = 9 \cdot 10^4$ MPa, $E'' = 4 \cdot 10^3$ MPa, $\nu' = 0.2$, $\nu'' = 0.35$, $\alpha' = 5 \cdot 10^{-6}$ deg^{-1}, $\alpha'' = 5 \cdot 10^{-5}$ deg^{-1}, $\sigma' = 2 \cdot 10^3$ MPa, $\sigma'' = 40$ MPa (σ' and σ'' are limits of ductility (strength for brittle materials) of the reinforcement and binder). Lay-up of the layers is carried out with $t_1 = 20°$C without prior tension of the reinforcement ($\sigma_0^m = 0$, $m = 1, 2, 3$) After polymerization of the binder at $t_2 = 170°$C the shell is cooled to $t_5 = 20°$C and it is removed from the mandrel. During the whole manufacturing process edges of the shell $r = r_0$, $r = r_1$ are free from external load: $T_{s(k)}(r_0) = T_{s(k)}(r_1) = 0$.

In order to estimate the level of residual structural stresses we introduce functions

$$k'_m = (\sigma'_{1(5)}/\sigma')_m \quad (m = 1, 2, 3),$$

$$k''_m = [(\sigma''_{1(5)})^2 - \sigma''_{1(5)}\sigma''_{2(5)} + (\sigma''_{2(5)})^2 + 3(\sigma''_{12(5)})^2]_m/\sigma''.$$

Presented in Fig. 1 are the results of calculating residual displacements u_5 (broken curves) and w_5 (solid curves) for a shell with parameters $a = 4.5$ m, $r_0 = 0.25$ m, $r_1 = 1.5$ m. Here and below lines 1 and 2 relate to values of shrinkage strains for the binder $\varepsilon_0 = 0$ and -0.005. Given in Figs. 2 and 3 are $k_m'(r)$, $k_m''(r)$ curves where solid, broken, and broken-dotted curves correspond to values $m = 1, 2, 3$. It can be seen from Fig. 3 that the level of shrinkage residual stresses in the binder is quite high and with $\varepsilon_0 = -0.005$ for layers with reinforcement angles $\psi = 0, 30, 60°$ it is 70, 86, and 88% of the critical value. For the minimum permissible value of formation pressure from (2.8) we find $p_*^+ = 7.3 \cdot 10^{-3}$ MPa. Whence it follows in particular that in the process in question it is possible to use vacuum forming. Curves in Figs. 1-3 corresponding to functions (curves 3) with $t_2 = 200°$C $\varepsilon_0 = -0.005$ illustrate the effect of polymerization temperature on residual displacements and stresses.

Given in Fig. 4 as a second example are the results of calculating residual deflection w_5 of the central surface (line 3) of a round cylindrical shell prepared by the wet winding method for glass tape K-115/100 on a steel mandrel in relation to force of tension N. The inner radius of the cylinder $r_1 = 0.03$ m, the outer radius $r_2 = 0.038$ m, polymerization temperature $t_2 = 70°$C, initial and final temperature of the manufacturing process $t_1 = t_5 = 20°$C, and the rest of the data for the calculation are taken from [9]. Given for comparison in Fig. 4 are values of residual deflections $w(r_1)$, $w(r_2)$, for the inner and outer surface of the cylinder (lines 1 and 4) calculated assuming smallness of stiffness of the semifinished product compared with the stiffness of the mandrel by the procedure in [9] where residual stresses and displacements in a round cylinder are found from solving the plane thermoelasticity problem. In calculating $w(r_2)$ it is assumed that in the heating stage for the semifinished product together with the mandrel deflection of its outer surface is mainly due

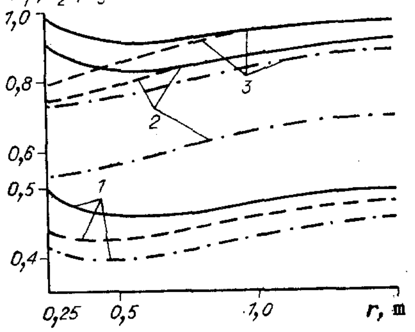

Fig. 3

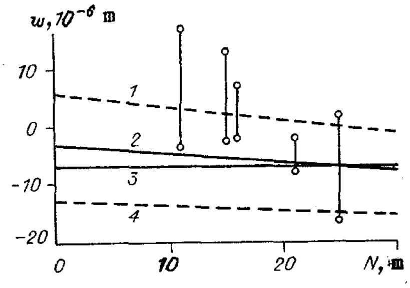

Fig. 4

to the amount of circumferential deformation for thermal expansion of the semifinished product. The correctness of this assumption follows from analyzing the solution in [9] if the considerable anisotropy of semifinished product material properties is considered. Extreme values of residual deflections of the inner surface obtained by experiment in [9] are shown by circles in Fig. 4. Straight line 2 relates to values of residual deflections for the central surface of the cylinder which is expressed in terms of residual deflection of front surfaces by the equation $w_* = 0.5\ (w(r_1) + w(r_2))$.

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DISLOCATIONS AND DISCLINATIONS IN NONLINEAR ELASTIC BODIES WITH MOMENT STRESSES

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UDC 539.3

A theory of dislocations and disclinations in elastic media which exhibit moment stresses and experience considerable strains is constructed. The marked effect of Volterra type dislocations in a Kosser nonlinearly elastic continuum is demonstrated by solving the problem of determining displacement and rotation fields in a multiconnected region with prescribed fields for the strain tensor and the bending strain tensor. Expression of Volterra dislocation characteristics in terms of the strain tensor field is given by means of a multiplicative contour integral. As a special case consideration is given to plane strain with which it is possible to delineate dislocations and disclinations in terms of normal contour integrals. Within the limits of moment nonlinear elasticity theory accurate solutions are found for the problem of screw dislocations and wedge dislocations. The effect of considering moment stresses and nonlinearity on the behavior of solutions close to the axis of a defect is analyzed.

1. In a model of a Kosser continuum [1-4] each particle of a solid has the degrees of freedom of an absolutely solid body. The position of particles in a deformed configuration is determined by radius-vector $\mathbf{R}$ and by strictly orthogonal tensor $\mathbf{H}$ called below the microrotation tensor. By using the principle of material indifference [5] it is possible to show that specific (per unit volume of reference configuration) potential energy W of an elastic Kosser continuum will depend on deformation of the body by means of two second rank tensors: tensor $\mathbf{U} = (\nabla^0\mathbf{R})\cdot\mathbf{H}^T$, called in the future the first measure of strain,

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